Notes on Exploration and Production Portfolio Optimization

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A New Era in Petroleum Exploration and Production Management

Exploration and Production (E&P) is the business of finding petroleum and getting it out of the ground. It is a risky business in that most exploration projects are total failures while a few are tremendously successful. Thus the best possible management of risk is crucial.

In the 1930's and 1940's, the development of seismic data collection and analysis substantially reduced the risk in finding petroleum. The resulting geology and geophysics (G&G) revolutionized petroleum exploration. Decision analysis has traditionally been applied to the information derived from G&G to rank projects hole by hole, determining on an individual basis whether or not they should be explored and developed.

Today this "hole-istic" approach is being challenged by a *holistic* one that takes into account the entire portfolio of potential projects as well as current holdings. This portfolio analysis starts with representations of the *local* uncertainties of the individual projects provided by the science and technology of G&G. It then takes into account *global* uncertainties by adding two additional G's: geo-economics and geo-politics. It thereby reduces risks associated with price fluctuations and political events in addition to the physical risks addressed by traditional G&G analysis. The holistic approach is based on but not identical to the Nobel-prize winning portfolio theory that has shaped the financial markets over the past four decades.

Portfolio thinking in petroleum E&P is based on understanding and exploiting the interplay among both existing and potential projects. It does not provide all the answers, but encourages E&P decision-makers to ask the right questions, such as:

- If we want a long-term expected return of, say, 15% on our investment, how can we insure against a cash flow shortfall over the first three years while minimizing our long-term risk?
- What should we pay for a new project, given the projects already in our portfolio?
- How would oil projects, as contrasted from gas projects, affect the impact of price uncertainty on my portfolio?
- What projects should we be seeking to reduce the effects of political instability in a given part of the world?
- What are the effects on financial risk and return if we insist on a minimum of, say, 40% ownership of any project?

As we shall see, analytical models can help in directly addressing these and similar questions once decision-makers in the petroleum industry become comfortable with the holistic perspective.

History

The Financial Markets

The story of quantitative portfolio analysis starts in the 1950s with the pioneering work of Harry Markowitz, who formalized the insight that increased return generally implies assuming increased risk. His "efficient frontier" of stock portfolios described the optimal investments for investors with differing aversions to risk. His 1959 book on *Portfolio Selection* [1], remains an excellent introduction to this subject. In the 1960's William Sharpe [2] streamlined and expanded on Markowitz's work with his Capital Asset Pricing Model (CAPM), while Franco Modigliani and Merton Miller 3] made other important contributions to the theory of valuation of securities. In the early 1970's Fischer Black, Myron Scholes [4] and Robert Merton [5] determined a rational pricing principle for stock options. All of the researchers mentioned received Nobel prizes in economics for their discoveries[1].

The above body of work changed the face of Wall Street forever, resulting in the widespread acceptance of an analytical approach to investing, and to the establishment of mutual funds, index funds, and derivative securities as common financial instruments.[2] Today, many elements of this revolution are already appearing in the energy markets.

Decision and Portfolio Analysis in Exploration and Production

The approach described in this paper is derived from two sources: portfolio theory, as described above, and decision analysis. Decision analysis was first applied to E&P on a project by project basis by C. J. Grayson [6] and G. M. Kaufman [7], and popularized by A. W. McCray [8], P. D. Newendorp [9], R. E. Megill [10 and 11], J. M. Cozzolino [12], and others. In 1968 David B. Hertz [13] discussed the application of the Markowitz model to risky industrial projects as opposed to stocks. In 1983 an author of the present article [14] proposed that petroleum E&P strategy be based on the Markowitz model, and discussed examples based on mainframe computations. In a follow-up article Ball [14] proposed the development of efficient frontiers for E&P projects using personal computers. Since 1990 the two authors of this article have collaborated on a sequence of models for consulting clients that have evolved to more closely reflect the differences between the stock market and E&P projects. Presentations to meetings of the Society of Petroleum Engineers by Lee Hightower [16] and R. A. Edwards and T. A. Hewett [17] have also reported on the application of financial portfolio theory to E&P. Hightower [18] authored a follow-up article in 1997. Also in 1997, Columbia University's Lamont-Doherty Earth Observatory founded a consortium of petroleum firms to share knowledge in the portfolio analysis of E&P projects [19]. The initial portfolio models used by this consortium, were developed by its director, John I. Howell, and were based on the approach of the authors of this present paper as described below.

The authors' approach deals with the following primary differences between stock and petroleum investments.

- Stock portfolios depend only on uncertain returns. E&P projects face both *local* uncertainties involving the discovery and production of oil at a given site, and *global* uncertainties involving prices, politics etc. Furthermore, uncertainties in stock returns usually follow a bell-shaped curve while E&P uncertainties are highly skewed and stress rare events.
- Risk in stock portfolios is usually measured in terms of volatility, the degree to which the portfolio swings in value. E&P portfolios must specifically track downside risk.
- The stock market is quite efficient[3] whereas the market for E&P projects is inefficient.
- E&P projects pay out over long time periods. Stocks can be bought or sold at will.
- A stock portfolio generally contains a small fraction of the outstanding shares of any one company. An E&P portfolio, on the other hand, often contains 100% of its constituent projects, creating budgetary effects.

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[1] The only exception being Black, who sadly passed away a few years before the prize was awarded for his work with Scholes.

[2] For an excellent and very readable history of this revolution in the financial markets, see *Capital Ideas* by Peter Bernstein [24].

[3] The term "efficient" is used here in its technical, economic sense. An efficient market is one in which there are no barriers to each item being priced at its actual value, as determined by all buyers and sellers, i.e., there are no "bargains."

This paper will show how we have dealt with each of the above issues. First a simple example will be presented to build intuition into portfolio analysis.

Retraining Our Intuition

Suppose you are responsible for investing $10 MM in E&P. Only two projects are available, either of which would require the full $10MM for a 100% interest. One is relatively "safe," the other relatively "risky"[4]. The chances of success are independent. This information is reflected in the Table 1.
This distribution of outcomes may also be displayed as histograms for each project as shown in Figures 1a and 1b. The expected net present value (ENPV) of each of the two projects can be shown to be the same as demonstrated in Equations 1 and 2.

$$ENPV_{Safe} = 60\%*\$50 + 40\%*(-\$10) = \$26 \text{ MM} \qquad \textbf{Equation 1}$$

$$ENPV_{Risky} = 40\%*\$80 + 60\%*(-\$10) = \$26 \text{ MM} \qquad \textbf{Equation 2}$$

To attach a concrete risk to the outcomes, imagine that if you lose money you will also lose your job. It is clear that you have only a 40% chance of unemployment with the safe project, and a 60% chance with the risky one. Since they both have ENPVs of $26MM, you cannot increase your ENPV by investing in the risky project. Therefore, if you had to choose between one or the other project, the obvious and correct answer is to invest the $10MM in the safe project.

The Diversification Effect

But suppose you could split your investment evenly between the two projects. Intuition cautions against taking 50% of your investment out of the safe project and putting it in the risky one. However, let us examine the four possible joint outcomes. Because the two projects are independent, the probability of any particular joint outcome is just the product of the probabilities of the associated individual outcomes. Note that the sum of the probability column must be 100% as we have exhausted all possibilities, as shown in Table 2. The histogram for the resulting portfolio is displayed in Figure 2.

This allocation of funds still provides an ENPV of $26MM. But now the only way you can lose money is with two dry holes. Since the projects are assumed to be independent, the probability of two dry holes is 40%*60% = 24%, cutting your risk of unemployment nearly in half! Intuition misleads: You can reduce risk by taking money out of a safe project, and putting it in a risky one. Although this answer is correct, it is not so obvious.

This is known as the diversification effect, popularly referred to as *not putting all your eggs in one basket*.[5]

One might consider this line of reasoning so fundamental that the petroleum industry would never do otherwise. However, the following summary of the conventional project selection process appeared in a December 1997 article in a leading professional petroleum exploration journal [20]: "Just rank exploration projects by expected present worth."

This suggests that one should rank projects from the best to the worst and then select projects from the top down until the budget is exhausted, ignoring the diversification effect. In the above example, this strategy

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[4] We shall have much to say about "risk" shortly. However, in every instance, we
factors: the probability of an undesired outcome, and how undesirable that outcome is.
[5] It is worth noting that the fact that the mass of the histogram of the 50/50 split has moved toward the center is related to the well-known central limit theorem of probability.

would have lead to allocating all of the funds to the "safe" project, which has the better risk reward ratio, but clearly this not the best portfolio.

The authors' experience with numerous E&P executives confirms that ranking "exploration projects by expected present worth" is the norm. A majority of those informally surveyed favor investing 100% in the "Safe" project. A few are aware that a diversification of a portion of the portfolio into "Risky" will actually reduce risk, but even these are not aware of how to arrive at an optimal mix.

Shooting the Moon

Some E&P executives feel that a major discovery every decade or so can sustain a large company. They might claim that they prefer to "Shoot the Moon," aiming for the highest possible expected return, regardless of the risk. The argument is that by ranking their projects in order of expected value, and then marching down the list until they run out of money, they will in fact arrive at the highest return/highest risk point the efficient frontier.

This is simply not true. For instance, the company could add projects by borrowing, which would have the effect of moving them to an even higher risk/higher return point than that achieved through ranking. Thus high-grading does not deliver the highest point on the efficient frontier at all, but results in an arbitrary and unexamined tradeoff between risk and return.

We are not arguing that a company should explore with borrowed money. We are claiming that high-grading ignores a universe of other portfolios, both riskier and safer, that can be illuminated through our approach.

Measures of Risk

Uncertainty is an inherent characteristic of our universe. Risk, on the other hand, is in the eye of the beholder. The naïve measure of risk we have used above (the probability of getting fired) was chosen here for illustrative purposes because it is easy to visualize and calculate. In practice, one would use a more sophisticated measure; for example, one that penalized larger losses more than small ones. However, diversification has a similar effect in reducing almost any sensible measure of risk. And, to the point here, full exploitation of the advantages of diversification are anything but intuitive. See Appendix 1 for a further discussion of risk measures.

Statistical Dependence and Its Sources

The above example assumes that the two projects are independent. In general, the interplay between two projects is more complex, in that their economic outcomes are interrelated. This is known as *statistical dependence*. The simplest type of statistical dependence is *correlation*, which comes in two varieties.

- ***Positive Correlation*** **-** A given outcome for one project ***increases*** the chance of an outcome in the same direction for the other. This diminishes the effect of diversification.
- ***Negative Correlation*** **-** A given outcome for one project ***decreases*** the chance of an outcome in the same direction for the other. This enhances the effect of diversification.

Let's consider the effects of positive correlation on a 50/50 split between safe and risky. We assume that the individual distributions of "safe" and "risky" remain as above. Now, however, in the event that "safe" succeeds, "risky" is more likely to succeed, and in the event that "safe" fails, "risky" is more likely to fail. Thus there is still a 40% chance that "safe" will fail, but if it does, the chance that "risky" fails is greater than 60%. Therefore the probability of losing your job is now greater than 24%. Next consider the effect of negative correlation. In the event that "safe" succeeds, "risky" is less likely to succeed, and in the event that "safe" fails, "risky" is less likely to fail. Thus there is still a 40% chance that "safe" will fail, but if it does, the chance that "risky" fails is less than 60%. Therefore the probability of losing your job is now less than 24%. It is important to note that correlation affects only the risks. The expected value of $26MM remains unchanged. In the portfolio approach, risk is managed by spreading the investments across a number of opportunities while avoiding positive correlations and seeking negative correlations. It is possible to do this optimally over numerous opportunities under diverse constraints, as the rest of the paper displays.

It is easy to understand the importance of correlation when one considers a fire insurance policy on a house. Since the insurance has a positive ENPV to the insurer, we know it has a negative ENPV to the insured, and is therefore a bad investment[6].

Statistical dependence may be due to many sources. The four listed below are not meant to be exhaustive, but are widely encountered in E&P projects.[7]

- Places
- Prices
- Profiles
- Politics

Places

The economic outcomes of two E&P sites in very close proximity (for example in the same field) will be positively correlated through geological similarities, and would not constitute a very diversified portfolio. On the other hand two sites in widely distant locations will display little or no geological correlation, and hence would be more diversified. "Places" can have corresponding implications for pricing (especially gas) and political issues as well as for geologic ones.

Prices

Petroleum projects produce crude oil and natural gas in various proportions. Crude oil prices generally track each other very closely worldwide. Thus, the economic outputs of oil projects worldwide are positively correlated relative to fluctuations in crude price. However, this is not true for natural gas. Natural gas prices in many parts of the world—notably in the United States—do not track either world crude oil prices or each other very well. Thus there would be a tendency for a portfolio consisting of a gas project and an oil project to be less positively correlated and therefore better diversified, relative to price, than a portfolio consisting of two oil projects.

As an example of this phenomenon, *The Wall Street Journal* reported in 1993 [21] that the economy of Houston, which had suffered during the crude oil price drop of 1986, had weathered a subsequent price drop successfully because it had diversified between oil and gas.

Profiles

A frequent concern is the timing of the flows of various elements of projects, which may extend for many years into the future. These flows might include such elements as cash flow, hydrocarbon production,

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[6] Unless your portfolio also includes a house (whose value is of course negatively correlated to the value of your insurance policy).

[7] Although we use "projects" throughout this paper, the same principles apply equally to E&P sites, exploration prospects, development projects, and/or acquisition properties. The point is that E&P companies can add reserves in three ways: exploration, development (and redevelopment) and acquisition. Each has a very different risk/reward profile, the *integrated* analysis of which could reveal innovative, optimal portfolios. Most E&P companies isolate these three functions in their respective functional silos and, at best, suboptimize each. [25]

reserve additions, and staff requirements. Often the more nearly constant these flows can be, the better. The correlation among these elements can be taken into consideration to minimize fluctuations in cash flows. These critical factors that can literally make or break a company can now be considered *and managed* explicitly.

As an example, consider how the correlations among project cash flows might be managed: Figure 3a shows the cash flow profile expected from two projects. If they were to comprise the portfolio, the resulting cash flow for the portfolio would be as shown in Figure 3b, with low valleys and high peaks. However, if two projects with expected cash flow profiles as shown in Figure 3c were to comprise the portfolio, the resulting cash flow for the portfolio would be as shown in Figure 3d. The peaks and valleys are leveled out—a much more desirable cash flow profile.

Politics

Petroleum investments have always been subject to political uncertainties, from the anti-trust decision against Standard Oil of 1911 through environmental regulations, to the Gulf War of 1991 and beyond. Projects subject to disruption in the same direction due to the same political event will be positively correlated. Negative correlation of projects may also be induced through political uncertainty. For example, consider two politically distinct regions that supply natural gas through two different pipelines to a single market. The political disruption of production in either of the two regions could lead to market shortages, and hence to increased prices and/or demands for the non-disrupted region. A portfolio consisting of one project in each negatively correlated region would thus be protected, or "hedged," against political risk in either region.

The Efficient Frontier

We have seen that a combination of a safe project and a risky project can be less risky than a pure investment in the safe project. This is displayed graphically in Figure 4. In this example, there were only two potential projects, and they had exactly the same ENPV. However, there will generally be projects of *various* expected returns and risks as depicted in Figure 5. By calculating the minimum risk portfolio for each of several levels of expected return, one can arrive at the curve shown in Figure 6. This curve shows the optimum trade-off between risk and return and is known as the *efficient frontier*.

Moving north from the efficient frontier increases risk without increasing expected return, while moving west decreases expected return without decreasing risk. Since each point on the efficient frontier has minimized the risk for that level of expected return, no portfolios exist to the southeast of the frontier. Thus the best portfolios are those on the efficient frontier itself. The concept of the efficient frontier and the method of finding it were the fundamental Nobel Prize-winning contributions of Markowitz in the 1950's. No rational person would wish to be at any point above the efficient frontier. But which point should you pick? That depends on your firm's willingness to suffer short-term volatility in the interest of long-term growth. In E&P one may also develop additional frontiers showing optimal trade-offs between reserve additions, budget level, cash flow shortfall, or other meaningful metrics.

Important Differences Between Stocks and Petroleum Projects

As mentioned earlier, the fundamental differences between stock returns and petroleum projects require modifications to the standard financial portfolio models.

Some primary differences in the underlying assets are:

- Types of Uncertainties
- Risk Measures
- Nature of Markets
- Timing Considerations
- Budgetary Effects

Table 3 reviews these in more detail.

E&P Portfolio Optimization Model (EPPO)

This section presents a simple E&P portfolio optimization model (EPPO), which unlike most standard financial portfolio models, can accommodate the characteristics of petroleum projects listed in Table 3. Excel versions of EPPO.xls may be downloaded from http://www-leland.stanford.edu/~savage or http://www.ziplink.net/~benball in formats for optimization with the Excel Solver, and What's*Best!*®.

As discussed earlier, the Markowitz and Sharpe models were intended for stock portfolios, and are not ideally suited to portfolios of petroleum projects[8]. EPPO is based on two technologies already in wide use individually in the petroleum industry: Monte Carlo simulation and linear programming. Here these technologies have been combined to create a single period stochastic linear program [22]. The advantages of this model for our purposes are:

- It allows for arbitrary realistic probability distributions of project outcomes as opposed to multivariate normal or log normal.
- It supports a wide variety of risk measures.
- It does not require historical data, but instead may be based on simulations, decision trees or other types of input
- It estimates not just the mean and variance of the portfolio, but the entire distribution of outcomes

EPPO works by feeding Monte Carlo trials into a linear program, which then finds a portfolio of minimum risk for a given expected NPV. This process is repeated for a range of desired NPVs, thereby determining the efficient frontier and the portfolios that comprise it.

The Spreadsheet

The basic elements of EPPO.xls are described in Figure 7. The program flow and underlying algebra are detailed in Appendix 2.

In the example shown here, the task is to optimize a portfolio from five exploration projects, A through E, given a particular budget.

Results

The results shown in Figures 8 a, b, and c were produced by running the EPPO model with a budget of $600, and successive required ENPVs of $1900, $2000, $2100, $2200, and $2300. Figure 8a shows the resulting risk/return trade-off curve running from an ENPV of 1900 and a Mean Loss[9] of 100 to an ENPV of 2300 and a Mean Loss of nearly 300. We refer to this curve as the Internal Efficient Frontier because it represents the best the firm can do with investments among its own projects. That is, each point on this efficient frontier represents the highest expected value at that level of risk, or, equivalently, the lowest risk for that expected value. There are no portfolios possible southeast of this frontier. All portfolios northwest of the frontier are inferior to a portfolio *on* the frontier, in that they offer an unnecessarily low return and/or an unnecessarily high risk.

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[8] For comparison, a spreadsheet version of the Sharpe model is included in the solver example file that ships with Microsoft Excel. A Markowitz model is included with INSIGHT.xla [26] See also www.AnalyCorp.com.

[9] Mean Loss, as defined and described in Appendix 1, is a particularly simple risk measure. Other risk measures, as also described in Appendix 1, may be used.

A firm not applying portfolio analysis would be unlikely to be on such an internal efficient frontier. We have denoted such a position by X in Figure 8a. Such a firm could increase its expected return at constant risk by moving to Z, or decrease its risk at constant return by moving to Y, or employ some intermediate strategy.

For each level of ENPV, the stacked bars of Figure 8b show the makeup of the efficient portfolio for each required ENPV. The vertical bars show the budget allocation to each of the five available projects in each efficient portfolio. Figure 8c combines both of these graphs to show a complete picture of the risk return trade-offs and the associated portfolios.

Management must choose such a point on the frontier, whereupon the underlying portfolio associated with that point is revealed. For example, assume Management decided that at a budget of $600, an ENPV of $2,200 is needed and a Mean Loss of $171 is acceptable. Then portfolio analysis would reveal that the budget should be allocated as follows to give the portfolio that would be most likely to yield that performance: 6% in Project A, none in Project B, 8% in Project C, 22% in Project D, and 64% in Project E.[10]

By contrast, current practice would be to decide separately on an investment level in each of the five projects, based largely on the intrinsic merits of each. The resulting portfolio would likely be northwest of the internal efficient frontier, like the point "X" in the Figure 8a. The total risk would be higher than necessary, or the expected value would be lower than necessary, or—most likely—both.

Generalizing to multiple time periods

One thing that EPPO and the Markowitz model do have in common is that both model *current* decisions only, based on potential future risks and rewards. A useful generalization would model both *current* and *future* decisions based on potential future risks and rewards. This would, in effect, be a marriage of portfolio theory and real options theory, and might be accomplished through either multi-period stochastic linear programming [22], or dynamic programming [23].

Business Implications: Asset Interplay Management

The fundamental point of view of this paper is that project by project or "hole-istic" analysis misses many important insights provided by the holistic perspective. The implications for the E&P business are that management must place at least as much emphasis on the interplay among projects as it does on the projects themselves. We refer to this as Asset Interplay Management, and believe that it is currently not adequately exploited by the industry. Table 4 shows some examples of how various types of corporate decision processes might be transformed by Asset Interplay Management.

It is tempting to think of Asset Interplay Management as just another analytical tool or computer program. The danger in this view is that the tool or program will be adopted, and won't deliver, thereby "inoculating" the company against a successful case of portfolio optimization for a generation of management. If the portfolio approach is to meet the expectations it is generating, top Management must understand that it represents a fundamentally new way of thinking about the business of E&P. This will require:

- Re-educating management, to develop and Asset Interplay Management until they become intuitive
- Re-structuring corporate systems to collect and interpret stochastic data from a global as well as local perspective

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[10] These "results" are no more than the results of a first iteration. For example, if 6% working interest in Project A is not practical, then the program should be rerun after placing the appropriate constraints on the working interest in Project A, e.g., ≥ 10%. Such iterations should be continued until all "results" are within the realm of practicality. This, then, will represent the optimum *practical* portfolio, which, of course, is the only one that matters in the real world.

- ❖ Revising reward programs to provide incentives for overall risk/reward positioning of the firm

Each of the above implications of Asset Interplay Management is valuable in itself, and offers insights not available through project-by-project analysis. Each avoids some of the subtle but systemic errors to which the industry is presently vulnerable. But these are just first steps as E&P enters the dawn of this new style of management.

At the beginning of this article, we posed several questions that Management should be asking, but cannot adequately be answered without Asset Interplay Management.

- ❖ If we want a long-term expected return of, say, 15% on our investment, how do we insure against a cash flow shortfall over the first three years?
  - ➢ This would require determining the optimum portfolio while constraining the first three-year cash flow to ≥ 0.
- ❖ What should we pay for a new project, given the projects we already have in our portfolio?
  - ➢ This could be determined by comparing the values of the portfolio with and without the new project, but *at constant risk*. (This is almost certain *not* to be the project's NPV.)
- ❖ How would oil projects, as contrasted from gas projects, affect the impact of price uncertainty on my portfolio?
  - ➢ Since Asset Interplay Management explicitly takes price interplay into account, the effect of each project on the portfolio's robustness relative to price instability can be determined.
- ❖ What projects should we be seeking to reduce the effects of political instability in a given part of the world?
  - ➢ Since Asset Interplay Management explicitly takes political interplay into account, the effect of each project on the portfolio's stability relative to political stability can be determined
- ❖ What are the effects on financial risk and return of insisting on a minimum of, say, 40% ownership of any project undertaken?
  - ➢ This would require determining the difference in portfolio value, *at constant risk*, with and without the 40% constraint.

Conclusions

- ❖ Portfolio principles developed for the financial arena must be modified before application to E&P.
- ❖ The portfolio perspective empowers decision-makers to focus on critical *business* issues, which guide asset interactions, viz., places, price, profiles, and politics.
- ❖ Portfolio management empowers decision-makers to *manage* risk, as well as measure it.
- ❖ Changes in perspective, intuition, and culture will do more to promote Asset Interplay Management than new computer programs.

Modern portfolio theory provided the conceptual underpinning for the financial engineering that now dominates Wall Street. How this will play out in the area of E&P remains to be seen. But one thing is certain: Asset Interplay Management offers new and powerful tools for dealing head-on with one of the elements which distinguishes the upstream business, but which too long has been handled only subjectively: RISK.

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Appendix A: Risk Measures

Variance

Variance (σ^2) has been the traditional measure of uncertainty both among statisticians and financial portfolio managers. The variance of an uncertain quantity is defined to be the average of the square of the deviation of the quantity from its mean. Because variance is measured in squared units (square dollars in the case of portfolio risk), it is also common to use the square root of variance or the standard deviation, σ as a measure of uncertainty.

Because variance measures the square of the deviation Δ from the mean it is a symmetric risk measure as shown in Figure A-1.

By its nature, the variance penalizes large deviations increasingly harshly, and, because it is symmetric, it penalizes upside deviations *equally* with downside ones.

When the underlying distributions of uncertainties are relatively symmetric as with stock prices, the variance is an appropriate measure of risk, see Markowitz [A-1] for example. However with asymmetric distributions such as the outcomes of petroleum projects, variance is not a desirable measure. For example the three projects shown in Table A-1 (taken from Schrage [A-2]) all have the same mean and variance, yet have obvious differences from a realistic risk perspective. This can be seen even more clearly by viewing the corresponding distributions, as shown in Figures A-2a, A-2b, and A-2c.

These projects are obviously quite different from each other. B, for example, has no chance whatsoever of loss. Yet for all three projects, the Mean = 10 and Variance = 400. Thus they would be indistinguishable using the mean and variance criteria customarily and appropriately used for stock portfolios.

Mean Absolute Deviation MAD

The Mean Absolute Deviation (MAD) is an alternative measure of risk [A-3] that is sometimes advantageous over variance for the following reasons:

- It may be applied directly to historical or Monte Carlo generated data regardless of distribution
- It may be minimized using Linear Programming
- It may be adapted to provide a wide variety of asymmetric measures of risk

MAD is calculated as follows. Suppose you had run M Monte Carlo trials of an uncertain outcome. We will define Δ_j to be the jth outcome minus the mean of all outcomes. Then the mean absolute deviation of the uncertainty is defined by Equation 3.

$$MAD = \frac{1}{M}\sum_{j=1}^{M}\left|\Delta_j\right| \qquad \textbf{Equation 3}$$

Notice that if we replaced the absolute value operator by the squaring operator we would be back to the variance.

A picture of the MAD risk function is shown in Figure A-3.
This penalizes deviations linearly. Notice that like the variance, the upside deviations are penalized equally with downside ones.

Mean Loss and other adaptations of MAD

A simple but useful adaptation of MAD, used in EPPO.xls, is **Mean Loss**, in which Δ measures only the deviation below 0 (not below the mean), as shown in Figure A-4. The mean loss of the three examples above is shown in Table A-2, clearly indicating that B is the least risky.

Mean loss is the average of one's losses. That is if you have a 50% chance of winning $1 or losing $1 the mean loss is 50¢. Mean loss has the desirable feature of distinguishing between upside and downside risk. However, like MAD it is a linear loss measure. That is, if you had a million dollars, and started losing money, mean loss would impose the same penalty on losing your 1st dollar that it does on losing your last dollar.

It is more realistic for the penalty to increase with deviation, as it does in the case of variance. The MAD model may be adapted further to model any piece-wise linear convex penalty $F(\Delta)$ such as that displayed in Figure A-5. Note that any number of straight-line penalty slopes and even a constraint on Δ may be imposed.

To create an LP model of this risk function, define two new LP variables for each scenario as follows, $y_{1j} \geq 0$ and $y_{2j} \geq 0$. These variables are constrained as shown in Equations 4, 5, 6, and 7.

$$y_{1j} \geq -\Delta_j - c$$ **Equation 4**

$$y_{1j} \leq d - c$$ **Equation 5**

$$y_{2j} \geq -\Delta_j - d$$ **Equation 6**

$$y_{2j} \leq e - d$$ **Equation 7**

For each scenario define F_j as in Equation 8.

$$F_j = a\, y_{1j} + b\, y_{2j}$$ **Equation 8**

Then the overall objective of the LP is to minimize the function defined in Equation 9.

$$\frac{1}{M}\sum_{j=1}^{M} F_j$$ **Equation 9**

In this way, customized risk measures may be constructed. One could even create a piece-wise linear approximation of the variance or semi-variance (squared deviation below the mean) if desired.

A customized risk measure suitable for production projects can easily be developed from the paradigm suggested in Figure A-5. In production projects, the danger is not so much an outright loss as it is an erosion of value. A risk parameter which measured Mean Value Erosion would simply involve setting "c" in Figure A-5 at a point *above* zero equal to the required economic threshold.

Appendix B: Notation and Algebraic Representation of EPPO

The Model

Table B-1 shows the notation used in EPPO.XLS model, while Table B-2 shows the model flow.

Algebraic Formulation

This EPPO.XLS model may be run with a variety of risk measures. As formulated here, it uses a measure we call mean loss, which is particularly easy to calculate and understand intuitively, but is not appropriate in all cases. The mean loss is the average of the economic losses over all Monte Carlo trials. Those trials without losses are averaged in as 0's. See Appendix 1 for a discussion of other risk measures and how to model them.

To model mean loss, we introduce m new variables y_i to record the loss under each Monte Carlo trial.

Then we minimize mean loss as shown in Equation 10, subject to the restrictions given in Equation 11, where the elements of the portfolio P are between 0 and 1.

$$MeanLoss = \frac{1}{m}\sum_{i=1}^{m} y_i$$ **Equation 10**

$$y_i \geq -P \cdot T_i, i = 1...m,\ P \cdot A \geq D, \text{ and } y_i \geq 0$$ Equation 11

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Tables

Table 1

	Outcome	NPV $MM	Independent Probability
Safe	**Dry Hole**	-10	40%
	Success	50	60%
Risky	**Dry Hole**	-10	60%
	Success	80	40%

Table 2

Outcomes of Investing 50% in Each Project
With Statistical Independence

	Safe	Risky	Probability	Return in $MM	Result
1	Success	Success	60% x 40% =**24%**	50% x $50 + 50% x $80 = **$65**	Keep Job
2	Success	Dry Hole	60% x 60% =**36%**	50% x $50 + 50% x (-$10) = **$20**	Keep Job
3	Dry Hole	Success	40% x 40% =**16%**	50% x (-$10) + 50% x $80 = **$35**	Keep Job
4	Dry Hole	Dry Hole	40% x 60% =**24%**	50% x (-$10) + 50% x (-$10) = **(-$10)**	Lose Job
	ENPV		24% x $65 + 36% x $20 + 16% x $35 + 24% x (-$10) = **$26**		

Table 3

Stock Portfolios | **E&P Projects**

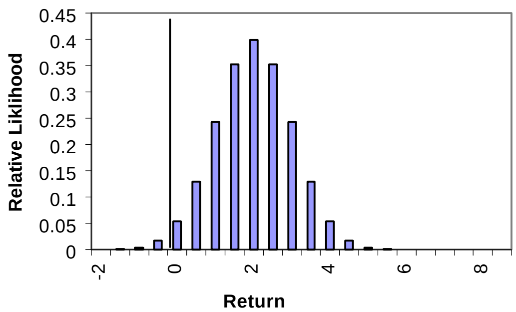
Types of Uncertainties

Stock Portfolios

a. Stock portfolio models are primarily based on price uncertainty.

b. The uncertainties of future stock returns are generally symmetric and bell shaped.

Distribution of a Stock Return
Mean Return = 2



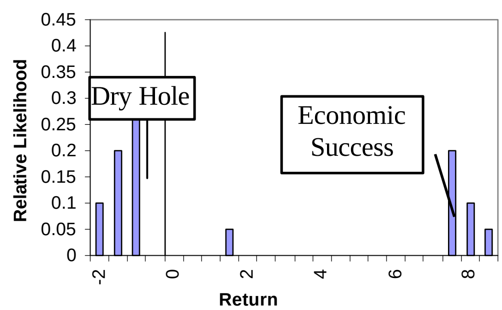
c. Estimates of distributions and statistical dependence are based at least in part on past history

E&P Projects

E&P projects face both *local* uncertainties involving the discovery and production of oil at a given site, and *global* uncertainties involving prices, politics etc.

The economic uncertainties of E&P are anything but normal.

Distribution of an E&P Project Return
Mean Return = 2



Distributions and statistical dependence of price information may be based on history. Other uncertainties must be modeled through decision trees or simulation.

Risk Measures

The risk experienced by an investor in the stock market is generally expressed in terms of the variance of the portfolio. This penalizes both upside and downside deviations equivalently, which makes sense for the symmetric distributions shown above.

Risk considerations for portfolios of petroleum investments depend on the concerns of management. Appropriate risk measures might be a cash shortfall in a given year, or a failure to achieve desired reserve addition by a given time. Because of the highly asymmetric distributions of outcomes, penalizing upside deviations does not make sense, and variance is not an appropriate risk measure.

Nature of Markets

Stock markets are quite efficient. One of the consequences is that the price one pays for a financial instrument is pretty much what it is worth. Therefore there are no bargains. In fact, many argue, with good reason, that one should not waste time designing stock portfolios and should invest entirely in index funds. These efficient markets give continuing feedback on values.

The market for petroleum E&P projects is not efficient. In addition, a project may have a very different value for one firm than it does for another. Hence there *are* "bargains" and "bad deals." Portfolio analysis is precisely the way to determine if a particular deal is good or bad for your firm. This inefficient market gives essentially no feedback on values.

Stock markets deal only with stocks and bonds.

Petroleum portfolios include currently owned assets, exploration prospects, development projects, and/or acquisition properties.

Timing Considerations

Stock portfolio analysis traditionally does not model time explicitly since stocks can be readily bought or sold at any time.

Petroleum investments have cash flows that play out over long periods of time. Therefore, time must be modeled explicitly.

Budgetary Effects

Stock portfolio models generally ignore the size of the budget. An efficient million-dollar portfolio is simply 1,000 times the size of an efficient thousand-dollar portfolio. Stock portfolios are concerned only with the proportions of various assets held, regardless of the size of the budget.

Petroleum portfolios are budget-dependent. Once one has taken a 100% interest in an E&P project, no more may be invested in that project. Therefore, different size budgets will have different proportions of the various projects in their respective optimal portfolios.

Table 4

Current Practice	Asset Interplay Management
1. Selecting a set of E&P projects to fund	
Rank (high-grade) the projects, start at the top of the list and go down until the budget is exceeded.	Select the set of projects that achieve optimal trade-offs of various risks and economic factors.
2. Determining long-term vs. short-term goals	
Establish long-term "return on investment"[11] and growth goals independent of their implications for the concomitant long-term risk or short-term volatility.	Make informed trade-offs between long-term goals for growth and the risks of short-term volatility, failure to meet reserve requirements or other measures of risk.[12]
3. Dealing with political and environmental risk	
Political and environmental risks are considered subjectively on a project by project basis.	Political and environmental risk of the entire portfolio is managed by taking the interplay among projects into account.
4. Evaluating a project for purchase or sale	
Determine the project's "market" value, that is, what other firms might pay for it, or base its value on an estimate of its ENPV.	Determine what the project is worth in the context of the firm's current holdings. Remember, I wouldn't spend a dime on a policy to insure *your* house, but an identical policy on *my* house is valuable to me, even though it has a negative NPV.
5. Determining the risk related cost of constraints and policies	
The costs of company policies and external constraints are appraised subjectively.	The implications of constraints and company policies can be evaluated on a risk/return basis.[13]

[11] Or return on capital employed, or similar metric.

[12] It is important to note that this approach captures the insights underlying utility theory, preference curves, etc., while avoiding the practical difficulties that arise from their explicit application. See Kenyon, Savage, and Ball [28].

[13] Constraints often reflect "strategic" issues of concern to top management, e.g., reserves replacement, cash flow for debt repayment, etc. Relative to stock market valuation, these may at times be as important or more important than ENPV. However, these constraints can be incorporated into the portfolio analysis and evaluated for their effects on risk and return. Brashear [25]

6. Determining Strategic Criteria for Future Exploration

Missing ingredients or strategic weaknesses are difficult to identify.

Missing ingredients or strategic thrusts that would contribute to the robustness of the portfolio are identified. The result is a shopping list of desirable qualities for additional projects or programs, given your firm's current portfolio and situation.[14]

7. Increasing the Value of the Firm

Sole focus is on expected net present value.

The focus of real E&P companies is never solely on expected net present value, but includes reserves replacement, debt ratios, cash flows, etc. At one extreme, risk is almost entirely ignored, but Froot, Scharfstein and Stein contend [27] that there are situations in which risk reduction can increase the value of the firm by assuring that cash flow is available when needed for critical investment.

Table A-1

Project	Outcome	NPV $MM	Probability	Mean	Variance
A	**Failure**	-10	50%	-10*.5+30*.5=10	$.5*(-10-10)^2+.5*(30-10)^2=400$
	Success	30	50%		
B	**Failure**	0	80%	0*.8+50*.2=10	$.8*(0-10)^2+.2*(50-10)^2=400$
	Success	50	20%		
C	**Failure**	-30	20%	-30*.2+20*.8=10	$.2*(-30-10)^2+.8*(20-10)^2=400$
	Success	20	80%		

[14] "Current holdings," of course, represents, the mass of most portfolios. "Hold and produce" takes little or no capital, requires no overt decisions, and yields great returns, especially if opportunity costs are ignored. However, serious consideration of the trading of current holdings (i.e., the sale of current assets in exchange for the purchase of new ones) is usually not seriously considered in any systemic way. However, such a study could significantly enhance the efficient frontier. In any event, a crucial point is that both current holdings and new projects opportunities must be evaluated together, as the portfolio's risk depends on the ways in which *all* of its constituent parts interact. Brashear [25]

Table A-2

Project	Outcome	NPV $MM	Probability	Mean Loss
A	**Failure**	-10	50%	10*.5 + 0*.5 = 5
	Success	30	50%	
B	**Failure**	0	80%	0*.8 + 0*.2=0
	Success	50	20%	
C	**Failure**	-30	20%	30*.2 + 0 *.8=6
	Success	20	80%	

Table B-1

Notation

n - Number of projects under consideration (5 in our example)

m- Number of Monte Carlo trials (40 in our example)

P - The portfolio. This is a vector of length n consisting of the working interest (between 0 and 100%) in each project. EPPO assumes that any working interest is possible, however, the model may be modified to force an "all or nothing" policy, or some other minimum or maximum working interest.

T_i - The i^{th} in the sequence of Monte Carlo Trials modeling the joint uncertain NPV's of the projects. T_i is a vector of length n.

A - The average NPV of each project over the m trials, a vector of length n.

$NPV_i(P)$ - The NPV of portfolio P under Monte Carlo trial i.

R(P) - A risk measure associated with portfolio P, calculated from $NPV_i(P)$, i=1..m. In EPPO, R(P) is the Mean Loss of P.

D- A desired level of ENPV

Table B-2

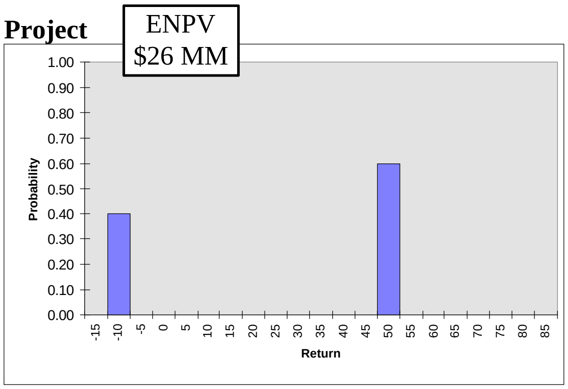
Model Flow

Monte Carlo Simulation	A Monte Carlo simulation of the joint economic outcomes of the projects is run, based on the local uncertainties, geo-technology and geo-science, and on the global uncertainties, geo-economics and geo-politics. The statistical dependence between projects must be preserved. The trials, Ti , i=1..m are stored.
Calculate ENPV and Distribution of NPVs	ENPV of the portfolio P is $P \cdot A$ = SUMPRODUCT(P,A) $NPV_i(P)$, the NPV of the portfolio P under the ith Monte Carlo Trial is $P \cdot T_i$ = SUMPRODUCT(P, T_i), i=1..m. The set of all $NPV_i(P)$, i=1..m provides an estimate of the probability distribution of the NPV for portfolio P. The chosen risk measure, R(P) is calculated from $NPV_i(P)$, i=1..m
	Choose a low level (D) of desired ENPV
Optimize	The linear program finds a portfolio P which minimizes the chosen risk measure R(P), subject to ENPV ≥ D
	If the linear program fails to find a solution, Halt. Otherwise, increase D and go back to previous step

Figures

Figure 1 a and b

Distribution of Outcomes of “Safe” Project



Distribution of Outcomes of “Risky” Project

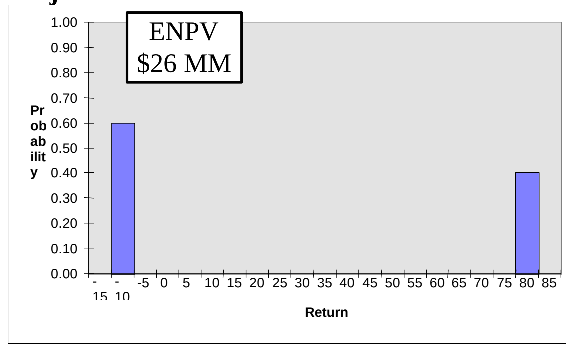


Figure 2

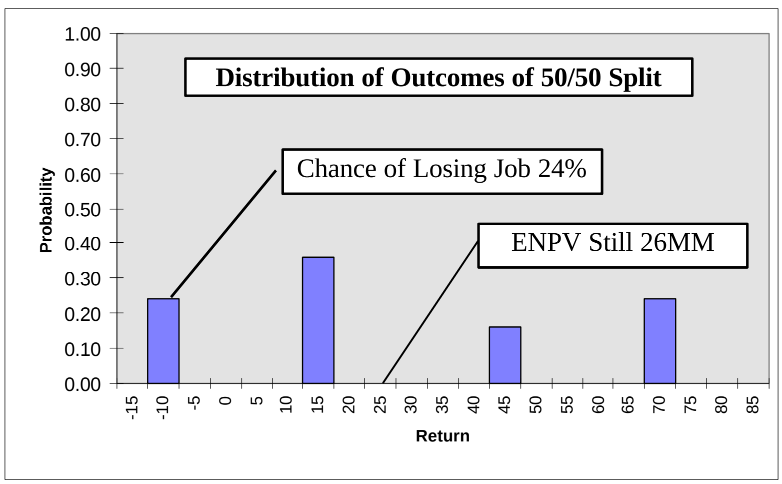
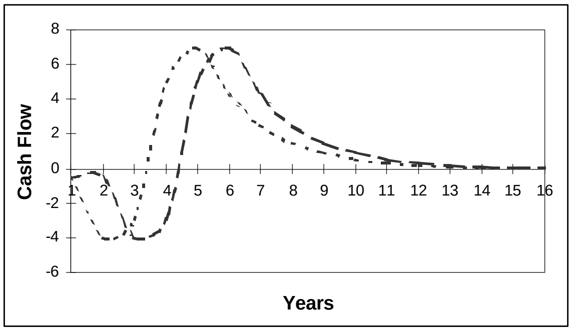
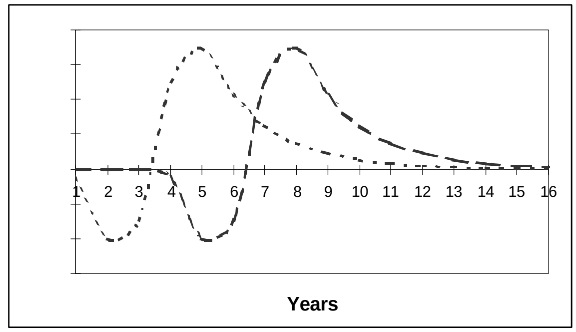


Figure 3 a, b, c, and d

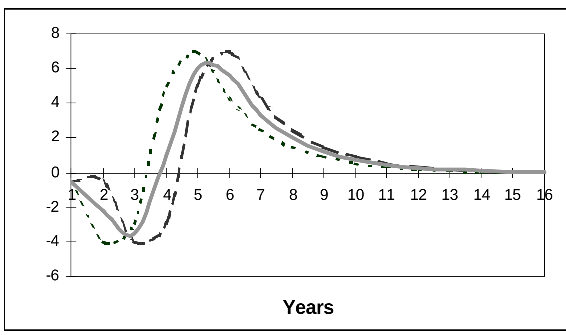
Projects with similar cash flow profiles will be positively correlated over time.



Projects with dissimilar cash flow profiles may be negatively correlated over time.



A portfolio consisting of a 50% share in each of the above cash flows (shown in solid line below) results in wide swings from negative to positive cash flow.



A portfolio consisting of a 50% share in each of the above cash flows (shown in solid line below) results in a relatively smoother cash flow.

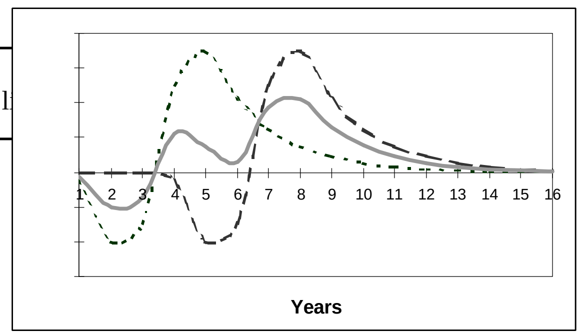


Figure 4

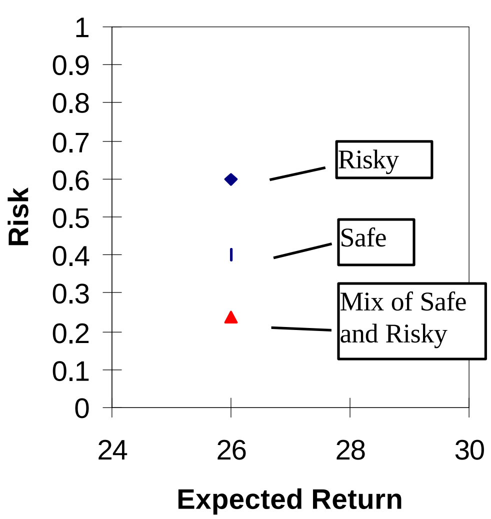


Figure 5

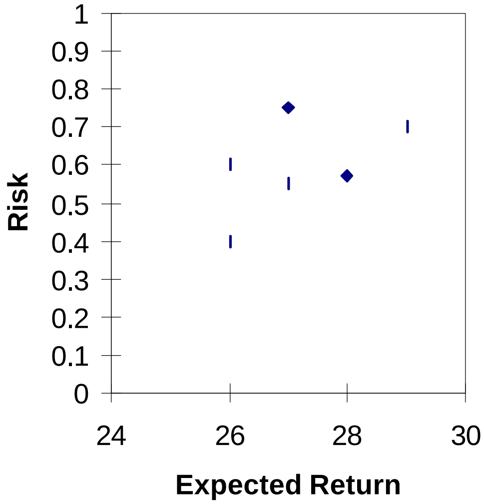


Figure 6

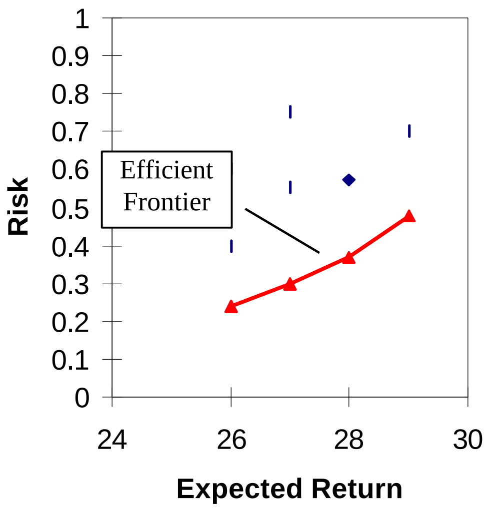


Figure 7

EPPO.XLS

Inputs

1. Monte Carlo trials of the economic outcomes of five projects designed to preserve the statistical dependence among projects. The user must provide this data for the projects under consideration.
2. Required ENPV
3. Budget.

Outputs

4. The Portfolio: percentages of each project undertaken, optimized to minimize the Mean Loss (5) for required ENPV (2) and Budget (3).
5. Mean Loss of portfolio.
6. Distribution of outputs for this portfolio.
7. Pie chart of % of budget in the various projects.

E&P Portfolio Optimizer EPPO

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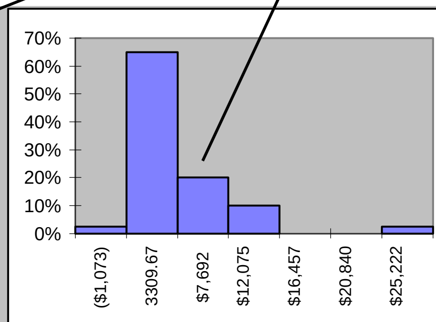
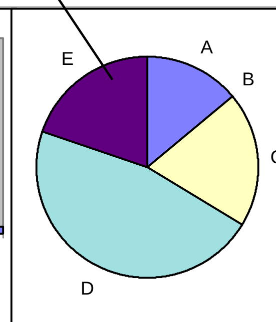
Portfolio Statistics

	NPV	Loss
Exp.	$2,500	$127
	>=	
Req.	$2,500	

	Cost	Reserves
Exp.	$791	484
	<=	
Req.	$1,100	





		Project A	Project B	Project C	Project D	Project E
	Portfolio	**30%**	**0%**	**41%**	**100%**	**41%**
	Project Statistics	**Project A**	**Project B**	**Project C**	**Project D**	**Project E**
Exp.	NPV	$820.39	$170.80	$890.51	$1,255.16	$1,562.84
Exp.	Cost	$372.69	$47.05	$374.08	$370.07	$383.88
Exp.	Reserves	161	32	218	240	262

Mean Loss calculations

Monte Carlo Generated Data

	Project A	Project B	Project C	Project D	Project E
Trial 1	$ 872.95	$ (8.00)	$ (100.00)	$ (300.00)	$ 23,147.60
Trial 2	$ 1,944.41	$ (47.00)	$ (100.00)	$ (250.00)	$ (350.00)
Trial 3	$ 1,910.78	$ (47.00)	$ (300.00)	$ 6,264.29	$ (350.00)
:	:	:	:	:	:
:	:	:	:	:	:
Trial 40	$ (393.00)	$ (47.00)	$ 1,441.28	$ (300.00)	$ (350.00)

Optimization Calculations

Portfolio by Scenario	Penalty		$127
$9,315	($9,315)	$0	$0
$142	($142)	$0	$0
$6,564	($6,564)	$0	$0
:	:	:	:
:	:	:	:
$36	($36)	$0	$0

Figure 8a

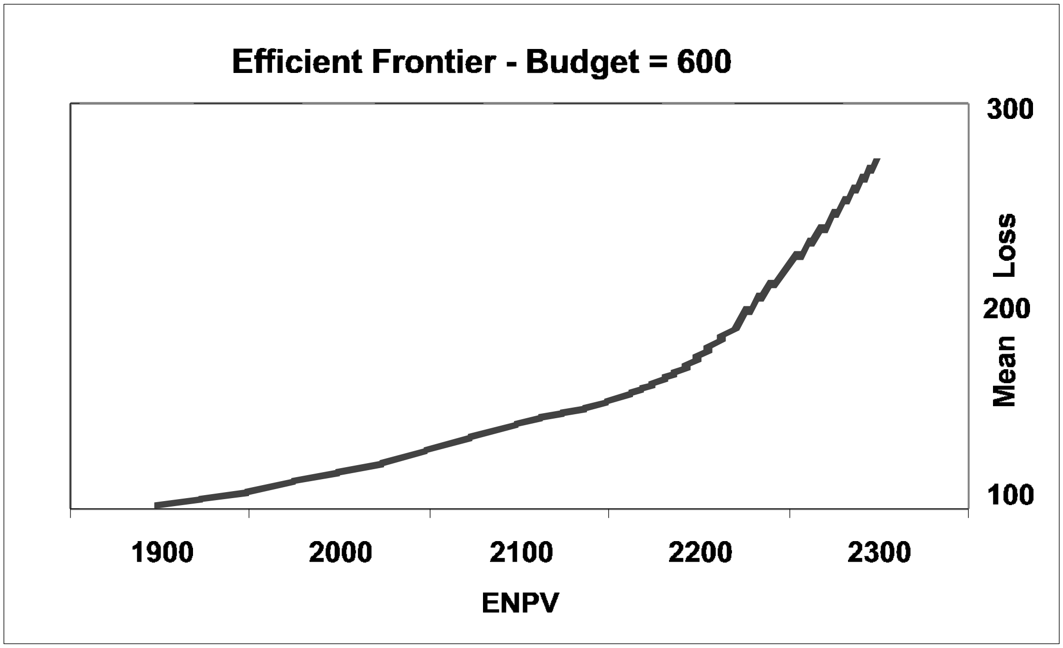

Efficient Frontier - Budget = 600
300
200
100
Mean Loss
1900
2000
2100
2200
2300
ENPV

Figure 8b

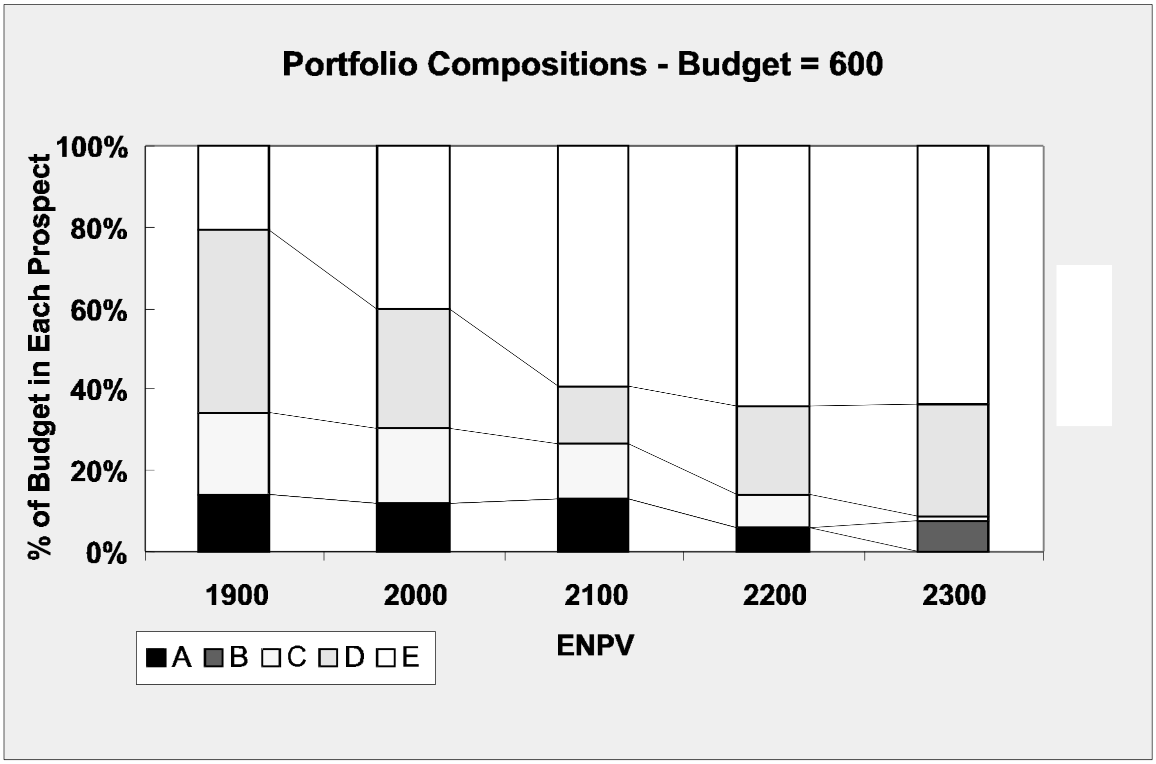
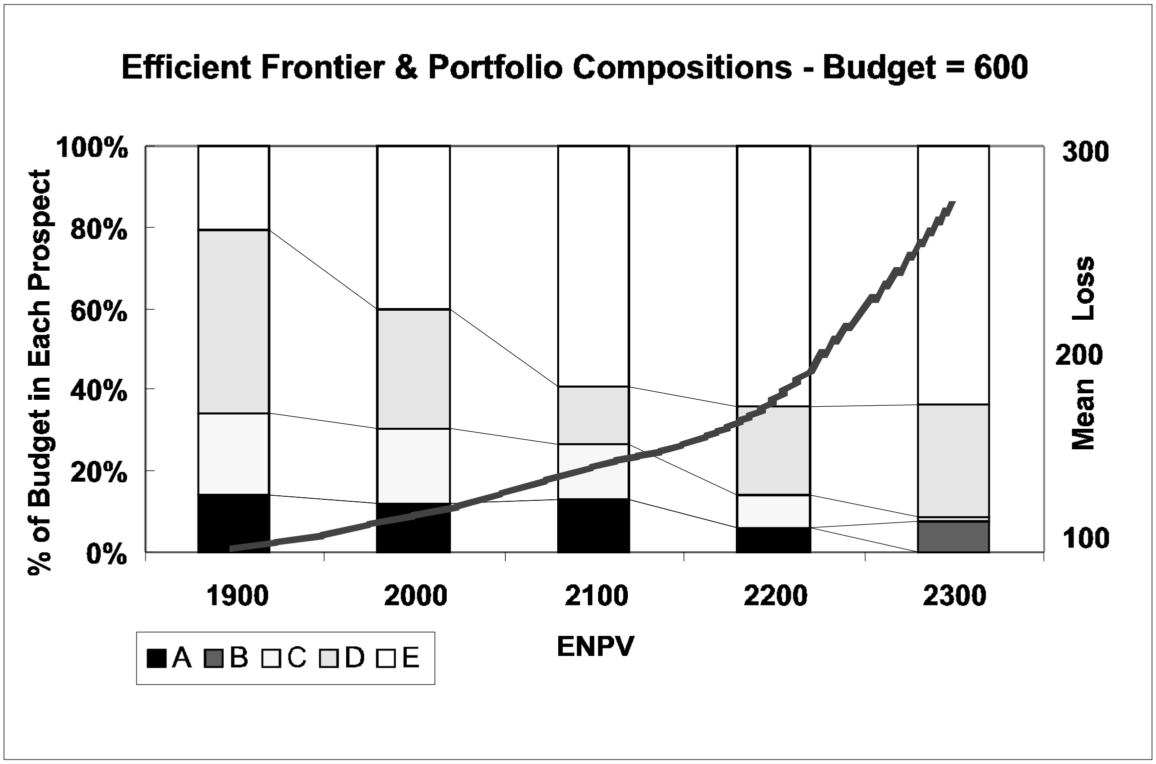

Portfolio Compositions - Budget = 600
% of Budget in Each Prospect
100%
80%
60%
40%
20%
0%
1900
2000
2100
2200
2300
ENPV
A
B
C
D
E

Figure 8c



Efficient Frontier & Portfolio Compositions - Budget = 600
% of Budget in Each Prospect
100%
80%
60%
40%
20%
0%
300
200
100
Mean Loss
1900
2000
2100
2200
2300
ENPV
A B C D E

Figure A-1

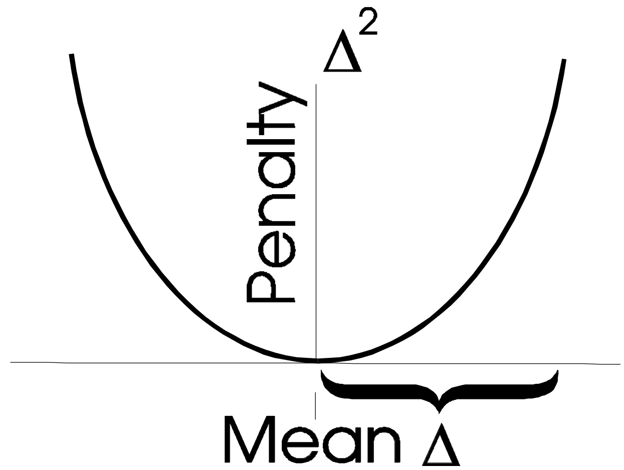
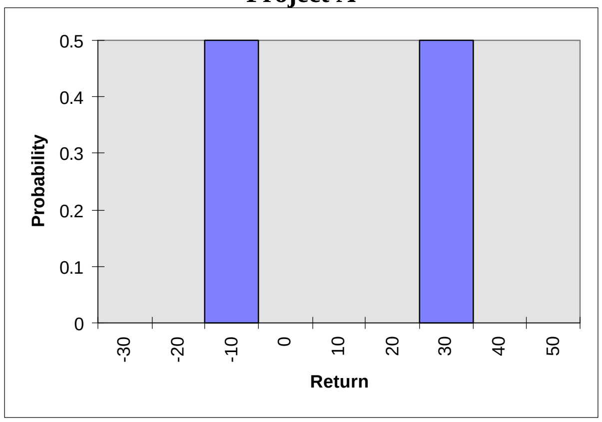
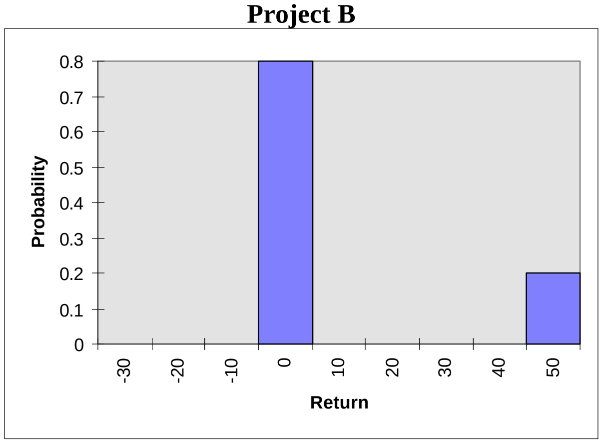


Figure A-2 a, b, and c

Project A



Project B



Project C

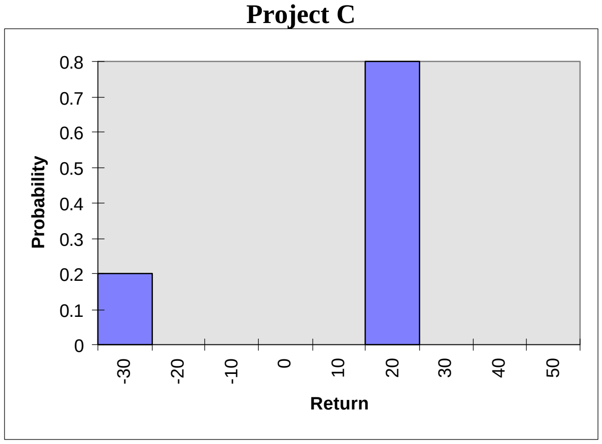


Figure A-3

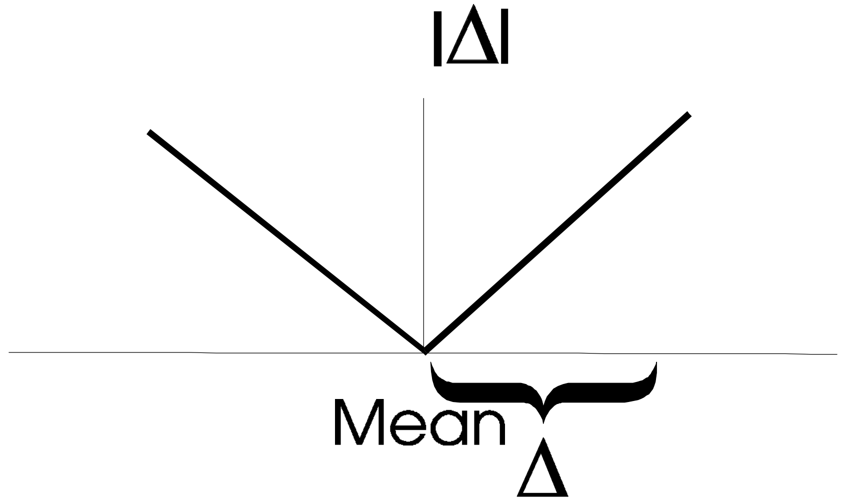


Figure A-4

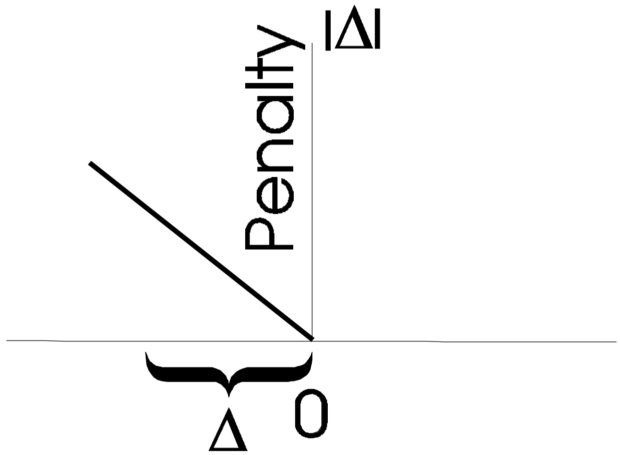


Figure A-5